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A note on Gaugino masses in Kaluza-Klein/radion mediated SUSY breaking

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Abstract: We review the equivalence of two approaches to study theories with gauge fields in extra spatial dimensions, namely the "$4D$" approach (with KK states) and the "$5D$" approach (with matching to the $4D$ theory at the compactification scale). In particular, we reiterate that there are two different power-law scalings of "effective" gauge couplings. In a supersymmetric framework with SUSY breaking in the radius modulus, i.e. the field which fixes the size of the extra dimensions, these two approaches seem to give gaugino masses at loop-level (with a possible enhancement due to large number of Kaluza-Klein states) [1], and tree-level [2], respectively. We show explicitly how this discrepancy can be resolved.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Supersymmetry Breaking, Supersymmetric Effective Theories.

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1. Introduction

There are two approaches to analysing theories with SM gauge fields in extra spatial dimensions [3]:

1. "$4D$" approach in which the extra dimensions "appear" in the form of Kaluza-Klein (KK) excitations of gauge fields. In this approach, the "effective" gauge coupling at energy scale E $(> R^{-1})$ is $N_{KK}(E) \times g^2_{4D}(E)$, where $N_{KK}(E) \sim R^{\delta} E^{\delta}$ is number of KK states lighter than E (including the zero-modes). Here δ is the number of extra dimensions and R is a typical size of an extra dimension. For a non-abelian gauge group, this effective coupling (i.e., number of KK states in loop growing with energy) results in g_{4D} running with power of energy [4].

2. "$(4+\delta)D$" or for short "$5D$" approach in which the gauge fields are treated as effectively being in $(4+\delta)D$ (non-compact) dimensions (above the compactification scale $\sim R^{-1}$) followed by matching to the (effective) $4D$ theory (at R^{-1}) given by $g^2_{4D} \sim g^2_{(4+\delta)D}/R^{\delta}$. In this approach, the $(4+\delta)D$ gauge coupling should "run" with power of energy since the gauge coupling is dimensionful and there is an integration over virtual extra-D momentum which results in a power-divergence.

Of course, these two approaches should be equivalent. However, in one example, this equivalence is not clear. Consider a supersymmetric version of this framework in which the radion, i.e. the scalar field which determines the size R of the extra dimensions and hence the mass $\sim n/R$ of the KK states, is part of a chiral superfield. Suppose F-component of the radion (chiral superfield) has a vev and thus breaks SUSY. In this case, as discussed recently, SUSY breaking is mediated to the MSSM gauginos (which propagate in the bulk) by the KK states[1] [1] (KK mediated SUSY breaking: KKMSB) or (what should be equivalent) by coupling of gauge fields to the radion [2] (radion mediated SUSY breaking: RMSB). This contribution to gaugino mass is determined by how the gauge coupling at low energies depends on R (i.e. the radion) — a priori, it is not clear that this dependence is the same in the two approaches. In fact, the $4D$ approach *as used in* [1] gives gaugino masses at loop-level, whereas [2] uses the $5D$ approach to obtain gaugino masses at tree-level — these two results are obviously different.

In this paper, we review the equivalence of the two approaches, in particular, the notions of "quantum" and "classical" power-law scalings of effective gauge couplings. This discussion is then used in the last section to clarify and resolve the discrepancy between the above two results for gaugino masses in KKMSB/RMSB. We show that, even in the $4D$ approach, there *is* a tree-level contribution to gaugino masses (corresponding to the tree-level effect in the $5D$ approach of [2]). This is due to the dependence of g_{4D} *at the cut-off* on R. However, this effect was *neglected* in [1]. Also, when the $(4+\delta)D$ theory is strongly coupled or (equivalently) when there are a large number of KK states (in the $4D$ approach), the loop contributions to gauge coupling (and to gaugino mass) studied in [1] (and *not* considered in the *tree-level* analysis of [2]) can be, *a priori*, *comparable* to tree-level terms. However, it turns out (as we show) that, even in this case, the net result for gaugino masses (i.e. combining tree- and loop-level effects) is well-approximated by the (tree-level) expression given in [2], provided $g_{4D} \lesssim \mathrm{O}(1)$ (as in the MSSM case).

2. Power-law scalings of gauge couplings

We begin with a discussion of two different kinds of power-law scalings for effective gauge couplings. Both scalings have the same origin — they are due to number of KK states increasing with energy (in $4D$ approach) or dimensionful gauge coupling and integration over extra-D momentum (in $5D$ approach). As we explain below, the terminology "quantum" or "classical" is used for this effect depending on whether KK states (or integration over extra-D momentum in $5D$ approach) renormalize (at loop-level) the gauge coupling or not.

[1] KK states have a non-supersymmetric mass spectrum since their masses $\sim n/R$ are determined by the radion.

2.1 "Classical" power-law scaling of gauge coupling

Consider U(1) gauge field ("photon") in bulk with all matter fields ("electron") charged under U(1) in $4D$ (i.e. on a 3-brane). Let us look at the effect in a tree-level process in the $4D$ approach, say, $e^+e^- \to \sum_{n,n'} \gamma^{(n)}\gamma^{(n')}$, where $\gamma^{(n)}$ denotes the KK state of the photon with momentum in the extra dimension (and hence mass) $\sim n/R$. For each n, n', $\sigma(e^+e^- \to \gamma^{(n)}\gamma^{(n')}) \sim e_{4D}^4/(16\pi)\, 1/E^2$ (as usual, where E is the c.m. energy) but the number of (kinematically accessible) final-states $\sim N_{KK}^2(E)$, where $N_{KK}(E) \sim E^\delta R^\delta$ is the number of KK states for each final-state photon. Thus, the total cross-section $\sim [e_{4D}^2 N_{KK}(E)]^2/(16\pi) 1/E^2$ can grow with power of energy for large δ.

This suggests that we can define an "effective" gauge coupling (as mentioned in the introduction) at energy $E(> R^{-1})$ which grows with power of energy:

$$e_{4D}^{\text{eff } 2}(E) \equiv e_{4D}^2 \times N_{KK}(E)\,. \tag{2.1}$$

Next, consider a loop-level process such as wavefunction renormalization of electron — at one-loop this also scales like a power of energy as follows. In the electron self-energy diagram, each photon KK state gives a log-divergent contribution (as usual from the $4D$ loop-momentum integration), but we have to sum over an infinite number of KK states so that the $4D$ theory with KK states appears non-renormalizable. We can introduce a cut-off Λ_{KK} to truncate the KK tower and get a renormalizable (i.e. with *finite* number of KK states) theory as an approximation — in this approximation, we can continue to use the $4D$ language of energy-*dependent* or running coupling/wavefunction. To compute the running electron wavefunction Z_e, we introduce the KK states as thresholds, i.e. we neglect the effect of KK states heavier than the renormalization group (RG) scale (as is usually done for other particle states). So, in this approximation, the effective coupling in the renormalization group equation (RGE) for Z_e at one-loop is (as above)

$$\frac{e_{4D}^{\text{eff } 2}(E)}{16\pi^2} = \frac{e_{4D}^2}{16\pi^2} \times N_{KK}(E) \tag{2.2}$$

instead of just $e_{4D}^2/\,(16\pi^2)$ (here E is the RG scale). Because of this effective coupling, the one-loop wavefunction renormalization of electron runs with power of energy.

Although there is power-law running of Z_e, running of e_{4D} is still logarithmic at one-loop (as in the case with photon in $4D$) — for simplicity, we will neglect this logarithmic ("mild") dependence of e_{4D} on energy in some cases (as in eq. (2.2)) and assume that e_{4D} is constant. This is because matter fields (electron) do not have KK states and so the coupling in vacuum polarization diagram for photon is still e_{4D}^2 and *not* $e_{4D}^2 \times N_{KK}(E)$. In other words, e_{4D}^{eff} does *not* contribute to the photon wavefunction, $\Pi^{\gamma\gamma}(q)$, and hence to e_{4D}, unlike the case of Z_e. Here $\Pi_{\mu\nu}(q) \sim (q_\mu q_\nu - g_{\mu\nu}q^2) \times \Pi(q)$, where q is the external ($4D$) momentum and $\Pi_{\mu\nu}(q)$

is the gauge boson self-energy. Because e_{4D}^{eff}, and hence processes involving the gauge coupling, acquire a E^{δ} dependence at *tree-level* (for example, $e^+e^- \to \gamma\gamma$ discussed above), whereas running of e_{4D} is *not* affected we call this a "classical" power-law scaling of gauge coupling, although it does effect Z_e at loop-level.

From $(4+\delta)D$ point of view, $e^2_{(4+\delta)D}$ (at the vertices in the electron self-energy diagram) has dimension of $(\mathrm{mass})^{-\delta}$ so that to obtain (dimension*less*) energy-*dependent*, i.e. running, wavefunction renormalization of the electron, we need to multiply by E^{δ} — in other words, by dimensional analysis, the RGE for Z_e is $d\left(\ln Z_e(E)\right)/d(\ln E) \sim 1/\left(16\pi^2\right) e^2_{(4+\delta)D} E^{\delta}$. Explicitly, there is an *extra* (compared to $4D$) *loop*-momentum integration for the photon (corresponding to extra-D momentum) which gives power-divergence. This implies that the $(4+\delta)D$ theory is non-renormalizable. Thus, there is also a contribution $\propto e^2_{(4+\delta)D}\ \Lambda^{\delta}_{4+\delta}$, where $\Lambda_{4+\delta}$ is the cut-off of the $(4+\delta)\ D$ theory. It is clear that this power divergence corresponds to infinite sum over KK states in $4D$ approach, i.e., $\Lambda_{KK} \leftrightarrow \Lambda_{4+\delta}$. On the other hand, at one-loop, $e_{(4+\delta)D}$ does not "run" like a power of energy since there is *no* extra-D momentum integration (for virtual electrons) in the vaccum polarization diagram for the photon. In this diagram, $e_{(4+\delta)D}$ at vertices is also dimensionful, but the dimension is "soaked" up by factors of R coming from the probability that the gauge boson propagating in the extra dimensions of size R is "near" the 3-brane (where the interaction with electrons takes place). Of course, the $4D$ loop-momentum integration for electrons results in the usual log-divergence.

In the process $e^+e^- \to \gamma\gamma$, in the $5D$ approach, we again have dimensionful couplings and so to get correct dimension for cross-section, we have to multiply by powers of energy $\sim \left(E^{\delta}\right)^2$ (in addition to those in a $4D$ calculation) — this corresponds to integration over *real* extra-D momentum (phase space) of each final-state photon.

2.2 "Quantum" power-law scaling of gauge coupling

This is absent for U(1) case. For non-abelian case (again with "quarks" in $4D$), in the $4D$ renormalizable approach mentioned above, the "gluon" wavefunction at one-loop has power-law energy dependence (unlike the photon wavefunction) due to number of KK gluons in the loop growing with energy (just like the electron wavefunction mentioned above), i.e., the coupling for the vacuum polarization diagram involving gluons in the loop *is* (effectively) $g^2_{4D} \times N_{KK}(E)$ instead of g^2_{4D} as in the photon case. In other words, g^{eff}_{4D} *does* contribute to $\Pi^{gg}(q)$ and hence renormalizes g_{4D}, unlike in the U(1) case. This implies that at one-loop $g^2_{4D}(E)$ "runs" with a power of energy [4].[2] We will show this RG calculation in the renormalizable approximation

[2] In the $4D$ theory with *finite* number of KK states (and with decoupling of KK states heavier than the RG scale), we can call the power-law energy dependence (at one-loop) of g_{4D} as power-law "running" (we already used this language for the electron wavefunction above), even though the fundamental $(4+\delta)D$ theory is non-renormalizable.

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in section 4. Because g_{4D}, and hence g_{4D}^{eff}, depends on a power of energy *at one-loop*, we refer to this effect as "quantum" power-law scaling of gauge coupling. To repeat, both classical and quantum power-law scalings of gauge coupling have the same origin — the number of KK states growing with a power of energy. The difference is that the former refers to power-law scaling of g_{4D}^{eff} at *tree-level*, whereas the latter refers to power-law scaling of g_{4D} or g_{4D}^{eff} at *(one)-loop level*. Also, the effective coupling in the RGE for quark wavefunction renormalization Z_q is $N_{KK}(E) \times g_{4D}^2(E)/\left(16\pi^2\right)$, where *both* N_{KK} and g_{4D} have power-law dependence on E.

In $5D$ approach also, (dimensionless) wavefunction renormalization of gluon from the vacuum polarization diagram,[3] and hence $g_{(4+\delta)D}$ (at one-loop), depends on a power of energy (unlike photon case). The reason is that $g_{(4+\delta)D}^2$ at the vertices is dimensionful and there *is* extra-D momentum integration for gluons (but not for quarks) in the loop which changes the usual ($4D$) log-divergence into a power-divergence (as in the case of Z_e).[4] By dimensional analysis, the RGE for quark wavefunction is $d\left(\ln Z_q(E)\right)/d\left(\ln E\right) \sim g_{(4+\delta)D}^2(E)\; E^{\delta}$ (up to a dimensionless loop-factor).

The above examples just illustrate the well-known facts that a $(4+\delta)D$ theory *with a cut-off scale* is equivalent to a $4D$ theory with a *finite* number of KK states below this scale and that the sum over KK states corresponds to integration over extra-D momentum.

3. Matching the $4D$ and $(4+\delta)D$ theories

On the basis of these arguments, we get the following plausible translation dictionary between the gauge couplings of the $(4+\delta)D$ theory and the $4D$ theory (with KK states), including *both* power-law scalings:

$$g_{4D}^{\text{eff}\,2}(E) \equiv g_{4D}^2(E) N_{KK}(E) \sim g_{(4+\delta)D}^2(E)\; E^{\delta}\,. \tag{3.1}$$

Of course, this is valid for $E > R^{-1}$. To repeat, $g_{(4+\delta)D}^2$ is dimensionful and hence by dimensional analysis we have to multiply by power of energy on the $(4+\delta)D$ side for "comparing" it to the $4D$ side; this power of energy corresponds to extra-D momentum integration (either in the loop while evaluating wavefunction renormalization or in an external leg as in the process $e^+e^- \to \gamma\gamma$). On the $4D$ side, N_{KK} (and hence $g_{4D}^{\text{eff}\,2}(E)$) grows with a power of energy which matches E^{δ} on the $(4+\delta)D$ side: we refer to this as classical power-law scaling. Thus, this relation is easily justified at tree-level (i.e. without E dependence in g_{4D} and $g_{(4+\delta)D}$) for both U(1) and non-abelian gauge groups (see the discussion of $\sigma\left(e^+e^- \to \gamma\gamma\right)$ and RGE for $Z_e(E)$ above). For the U(1) case, due to matter fields on a 3-brane, there is the usual logarithmic running of e_{4D} and we also expect logarithmic dependence on energy in $e_{(4+\delta)D}$. Thus,

[3] i.e. $\Pi^{gg}(Q)$, where Q is the external ($(4+\delta)D$) momentum.

[4] $\Pi^{gg}(Q)$ also has a contribution $\propto g_{(4+\delta)D}^2\; \Lambda_{4+\delta}^{\delta}$.

in the U(1) case, the above relation is fairly accurate at the quantum-level also (i.e., *including* the logarithmic (mild) energy dependences in e_{4D} and $e_{(4+\delta)D}$).

Furthermore, in the *non-abelian* case, at the quantum-level, g_{4D} runs like a power of energy (quantum power-law scaling) as mentioned earlier and as will be shown explicitly by a calculation in the renormalizable approximation (see section 4). As argued earlier, $g_{(4+\delta)D}$ also has (loop-suppressed) power-law dependence on E and this dependence should correspond to the power-law running of g_{4D}. Thus the above relation is plausible in the non-abelian case at the quantum level also, i.e. *including* the (power-law) energy dependences in g_{4D} and $g_{(4+\delta)D}$ (see the discussion of RGE for $Z_q(E)$ above). We can then say that $g_{(4+\delta)D}$ also "runs" (at one-loop) like a power of energy, even though, as mentioned earlier, the $(4+\delta)D$ theory is non-renormalizable and so $g_{(4+\delta)D}$ does not run in the $4D$ sense. In other words, by dimensional analysis, the *one-loop* "RGE" for $g_{(4+\delta)D}$ is $dg^{-2}_{(4+\delta)D}(E)/d(\ln E) \sim E^\delta$ (up to a dimensionless loop-factor). The $5D$ approach, i.e. the analysis with the dimension*ful* coupling $g_{(4+\delta)D}$, is similar to the discussion in [5] of wilsonian RGE's in $4D$ with *non*-renormalizable (irrelevant) operators. To prove this correspondence between (one-loop) running of g_{4D} and that of $g_{(4+\delta)D}$, and hence the above relation in the non-abelian case, we would have to compute explicitly the loop correction in $(4+\delta)D$ which will not be attempted here.

From eq. (3.1) and using $N_{KK}(E) \sim R^\delta E^\delta$, we get the matching condition valid at *all* energies above R^{-1}:

$$g^2_{4D}(E) \sim \frac{g^2_{(4+\delta)D}(E)}{R^\delta}\,. \tag{3.2}$$

A related (and the usual) way to derive this matching is to do a KK decomposition of the canonically normalized $(4+\delta)D$ gauge field, i.e. with action $S = \int d^4x d^\delta y\ F^2_{\mu\nu} + \int d^4x\ \bar{\psi}(x)\gamma^\mu A_\mu(x, y = 0)\psi(x)\ g_{(4+\delta)D} + \cdots$, where y denotes the extra dimensions and we have assumed that the matter field ψ is localized at $y = 0$: the gauge boson in $(4+\delta)D$ has mass dimension $1+\delta/2$. The zero-mode of the KK decomposition, $A^{(0)}_\mu(x)$ with mass dimension 1, is the usual $4D$ gauge field and its wavefunction has a normalization factor (from the volume of the extra dimensional space) $\sim 1/\sqrt{R^\delta}$ so that the coupling of zero-mode gauge boson to matter field is $\bar{\psi}\gamma^\mu A^{(0)}_\mu\psi\ g_{(4+\delta)D}/\sqrt{R^\delta}$ and hence we get the above result.[5] Thus, this argument justifies the above matching relation (eq. (3.2)) at *tree*-level (i.e. *without* the energy dependence). We claim that this relation is valid at the *quantum*-level.

[5]In a supersymmetric theory with holomorphic normalization, the action is $S = \int d^4x d^\delta y\ 1/g^2_{(4+\delta)D} F^2_{\mu\nu} + \int d^4x\ \bar{\psi}(x) A_\mu(x, y = 0)\psi(x) + \cdots$, where the gauge boson has mass dimension 1 and $g^2_{(4+\delta)D}$ has mass dimension $-\delta$. Since the zero-mode is the Fourier component of $(4+\delta)D$ gauge field which is a constant function of the extra dimensional coordinate, integration over the coordinate of the extra dimension (to get $4D$ action) gives simply a volume factor in the kinetic term for the zero-mode, i.e. $S = \int d^4x[R^\delta/g^2_{(4+\delta)D}(F^{(0)}_{\mu\nu})^2 + \bar{\psi}\gamma^\mu A^{(0)}_\mu\psi + \cdots]$. Hence we get above relation for the (holomorphic) gauge couplings.

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4. Strong coupling and need for a cut-off

We now review the relationship between the compactification scale and the strong coupling scale in these two approaches.

Consider U(1) case where e_{4D} has a logarithmic (mild) energy dependence. We see that e_{4D}^{eff} (eq. (2.2)) reaches strong coupling, i.e. $e_{4D}^{\text{eff}\,2}/\left(16\pi^2\right) \sim \mathrm{O}(1)$ (so that loop corrections become $\sim \mathrm{O}(1)$ or $\sim$ tree-level terms) at an energy M such that $M^\delta \sim R^{-\delta} \times \left(16\pi^2\right)$ (assuming $e_{4D} \sim \mathrm{O}(1)$ at R^{-1}).

From $(4+\delta)D$ point of view, the gauge theory is non-renormalizable (gauge coupling is dimensionful and so there are power divergences in, say, electron wave-function renormalization as shown above) and so we need a cut-off, say M. The value of $e^2_{(4+\delta)D}$ at strong coupling (i.e. the maximum value for $e^2_{(4+\delta)D}$) is $\sim l_{4+\delta}/M^\delta$, where $l_D \equiv 2^D \pi^{D/2} \Gamma(D/2)$, such that $1/l_D$ is the loop expansion parameter (loop-factor, for short) in D dimensions (for example, $1/\left(16\pi^2\right)$ in $4D$) [6]. Thus, the maximum value for e^2_{4D} (from eq. (3.2)) is[6] $\sim l_{4+\delta}/\left(R^\delta M^\delta\right)$ and so if we require $e_{4D} \sim \mathrm{O}(1)$, then we need $M^\delta \lesssim l_{4+\delta}\, R^{-\delta}$, i.e., in agreement with above, we see that that we cannot (perturbatively) extrapolate the theory to energies larger than $\sim$ (1/loop-factor $\times\, R^{-1}$).

The non-abelian case is a bit subtle since g_{4D} (or $g_{(4+\delta)D}$) also (in addition to $N_{KK}(E)$) has a power-law dependence on E due to running (at one-loop). In fact, with no matter fields in bulk, this effect *decreases* g_{4D} at higher energies (as shown below) and thus competes with N_{KK} in determining how g_{4D}^{eff} depends on energy (see $4D$ side of eq. (3.1) and below).

We now calculate the power-law running of g_{4D}. In the renormalizable approximation, we have a finite number of KK states (up to a cut-off Λ) and we can treat the KK states as thresholds for running of couplings, i.e., in the RGE, we decouple the KK states at their masses $\sim n/R$. Thus, at one-loop, we get the following RGE, neglecting the effects of the zero-mode of the gauge field and matter fields:

$$\frac{\partial g_{4D}^{-2}(E)}{\partial \ln E} \approx -\frac{b_{KK}}{8\pi^2} N_{KK}(E)\,, \tag{4.1}$$

where $N_{KK}(E) \sim E^\delta R^\delta$ is the number of KK states lighter than E (*excluding* the zero-modes) and $b_{KK} < 0$ is the β-function coefficient for KK states at each (massive) level. To be precise, let us assume that the δ extra dimensions are compactified on circles of equal radii R so that the mass splitting between KK states is $\approx 1/R$. We also assume $E \gg 1/R$ so that the sum over KK states can be approximated by an integral. Then, $N_{KK}(E)$ is given by the volume of δ-dimensional sphere of radius ER (which is the maximum quantum number of the KK states), but *not* counting the zero-mode, i.e.

$$N_{KK}(E) \approx \hat{V}_\delta (ER)^\delta - 1\,, \tag{4.2}$$

where $\hat{V}_\delta = 1/\delta \times 2\pi^{\delta/2}/\Gamma\left(\delta/2\right)$ is the volume of a unit-sphere in δ dimensions.

[6] In the U(1) case, $e_{(4+\delta)D}$ does not run and so its value at the compactification scale $\sim R^{-1}$ is the same as at M.

Integrating the RGE in eq. (4.1) (using eq. (4.2)) from R^{-1} to energy scale E ("bottom-up" calculation as in [4]), we get the gauge coupling at E in terms of the gauge coupling at R^{-1}:

$$\begin{aligned} g_{4D}^{-2}(E) &\approx g_{4D}^{-2}(R^{-1}) - b_{KK}/8\pi^2 \left[\left((ER)^\delta - 1 \right) \hat{V}_\delta/\delta - \ln(ER) \right] \\ &\approx g_{4D}^{-2}(R^{-1}) - b_{KK}/8\pi^2 \left[N_{KK}(E)/\delta + (1-\hat{V}_\delta)/\delta - \ln(ER) \right]. \end{aligned} \quad (4.3)$$

We see that g_{4D} *decreases* (at one-loop) with a power-law as the energy is increased as expected since the (massive) gauge field KK states make the theory *more* asymptotically free. Thus, the effective gauge coupling is

$$\frac{g_{4D}^{\text{eff}\,2}(E)}{16\pi^2} \approx \frac{g_{4D}^2(R^{-1})/16\pi^2 \times N_{KK}(E)}{1 - b_{KK}/8\pi^2 \left[N_{KK}(E)/\delta + (1-\hat{V}_\delta)/\delta - \ln(ER) \right] g_{4D}^2(R^{-1})}. \quad (4.4)$$

We can trace the $\ln(ER)$ factor in the equations above to the fact that the zero-mode of the gauge field does *not* have the β-function coefficient b_{KK}. As mentioned earlier, we neglect the effect of the zero-mode and also of matter fields — strictly speaking there should be an additional term $\propto b_0 \ln(ER)$ in the above equations, where b_0 is the β-function coefficient of zero-modes (zero-mode of gauge field + matter fields), such that if $b_{KK} = b_0$, then these two $\ln(ER)$ terms cancel each other. As mentioned before, the expression for $N_{KK}(E)$ in eq. (4.2) and hence the solution to the RGE is really valid only for $ER \gg 1$, in which case the $(ER)^\delta$ factor dominates the $\ln(ER)$ factor in the above equations and hence the latter can be neglected. Of course, the zero-modes also renormalize g_{4D} below R^{-1} as usual.

From eq. (4.4) (and using (4.2)) we see that at an energy M given by $M^\delta \sim 8\pi^2/g_{4D}^2(R^{-1})R^{-\delta} \times -\delta/b_{KK}$, the power-law term $\propto N_{KK}$ (from the running of g_{4D}) starts to dominate in the *denominator* in eq. (4.4) (i.e. it becomes O(1)) which implies that for $E \gtrsim M$ the power-law running of g_{4D} "cancels" the power-law energy dependence of N_{KK} in the *numerator* in eq. (4.4). But, we see that at $E \sim M$, $g_{4D}^{\text{eff}\,2}/\left(16\pi^2\right) \sim O\left(-\delta/(2\, b_{KK})\right)$ and also that $g_{4D}^{\text{eff}\,2}/\left(16\pi^2\right)$ reaches a *constant* value, $-\delta/(2\, b_{KK})$, as $E \to \infty$. Thus, by the energy scale M at which $g_{4D}^{\text{eff}\,2}$ starts "leveling" off, we see that it has already reached strong coupling. In other words, even though $g_{4D}^{\text{eff}\,2}$ does *not* grow "indefinitely" with energy (unlike in the U(1) case: eq. (2.2)), the theory becomes non-perturbative above $M \sim R^{-1} \times (1/\text{loop-factor})^{1/\delta}$ as in the U(1) case.[7]

[7] At strong coupling, one might worry about *(power-law) higher*-loop corrections to the gauge coupling and thus whether the one-loop RGE for the gauge coupling suffices. However, if the massive gauge KK states form $N = 2$ SUSY vector multiplets (as in [4]), then there are *no* corrections (power-law or logarithmic) beyond one-loop involving *only massive* gauge KK states. Of course, at two-loop, the gauge coupling depends on the wavefunction renormalization of matter fields which are on 3-branes (and also on wavefunction renormalization of *zero*-mode gauge fields which do *not*

To complete this discussion, we have to look at the relation between the strong coupling scale and the compactification scale in the non-abelian case *from the* $(4+\delta)D$ *point of view.* The argument is similar to that in the U(1) case, except that we have to run $g_{(4+\delta)D}(E)$ from the cut-off (M) to R^{-1}[8] and then match to the (effective) $4D$ theory to give $g_{4D}(R^{-1})$. Although, as before, we refrain from doing this $(4+\delta)D$ calculation, it should agree with the above calculation in the $4D$ theory with KK states.

5. Gaugino masses in Kaluza-Klein/radion mediated SUSY breaking

Next, we use the discussion in the previous sections to resolve the discrepancy in the results for gaugino masses in KKMSB [1] and RMSB [2].

Suppose SUSY is broken by the radion — to repeat this is the field whose vev determines the size R of the extra dimension(s). In general, to compute (zero-mode) gaugino masses, we have to determine how the low energy ($4D$) gauge couplings (in other words, the wavefunction renormalization of gauge fields) depend on the SUSY breaking modulus [7]. Thus, in this case, we need to integrate the RGE (eq. (4.1)) starting from cut-off Λ ("top-down" approach; see, for example, [8, 9]) and compute the dependence of $g_{4D}(R^{-1})$ on R — this just amounts to setting $E \approx \Lambda$ in eq. (4.3) and rewriting it to get

$$g_{4D}^{-2}(\mu \sim R^{-1}) \approx g_{4D}^{-2}(\Lambda) - \frac{b_{KK}}{8\pi^2}\left[\frac{\hat{V}_\delta}{\delta}\left(1-(\Lambda\, R)^\delta\right) + \ln(\Lambda R)\right] \tag{5.1}$$

with

$$g_{4D}^{-2}(\Lambda) \approx g_{(4+\delta)D}^{-2}(\Lambda)\,(2\pi R)^\delta \tag{5.2}$$

obtained from the matching condition, eq. (3.2) (we have added factors of 2π in the extra dimensional volume). Here, we have neglected the effects of the zero-mode of the gauge field and also of matter fields (which are asssumed to be on 3-branes) on the running. The above result is the same as eq. (32) in [9]. As mentioned earlier, the solution in eq. (5.1) is strictly speaking valid only for $\Lambda R \gg 1$; in this case, the

form $N=2$ SUSY vector multiplets) which, in turn, evolves like a power of energy due to the KK states of gauge fields (as discussed earlier). However, this *two-loop* power-law running of the gauge coupling is clearly suppressed by the usual loop-factor $\sim g_{4D}^2/\left(16\pi^2\right)$ compared to the *one-loop* power-law running. Thus, the one-loop RGE for the gauge coupling suffices (for our purpose), even at strong coupling. Of course, wavefunction renormalization of matter fields gets a contribution from the massive KK states to *all* loop orders and hence, at strong coupling, the one-loop RGE will *not* suffice for evaluating it, i.e. the n-loop term $\sim \left[N_{KK}\ g_{4D}^2/\left(16\pi^2\right)\right]^n$ is O(1) at strong coupling.

[8] The reader might still be uncomfortable with the terminology "running" of gauge coupling in $(4+\delta)D$ — in that case, a better term is "finite energy-dependent corrections" to the gauge coupling.

$\ln(\Lambda R)$ term can be neglected compared to the $(\Lambda R)^{\delta}$ term. Although, the expression for $g_{4D}\left(\mu \sim R^{-1}\right)$ in eq. (5.1) has been obtained using the $4D$ approach, it is clear from the discussion in the previous sections that the $5D$ approach will give the same expression (with $g_{4D}^{-2}(\Lambda)$ given by eq. (5.2)).

The gaugino mass is given by

$$M_{\tilde{g}}(\mu \sim R^{-1}) \approx F_T \; g_{4D}^2(\mu \sim R^{-1}) \frac{\partial g_{4D}^{-2}(\mu \sim T)}{\partial T}\bigg|_{T \sim R^{-1}} , \tag{5.3}$$

where T is the canonically normalized radion chiral superfield, i.e. $\langle T \rangle \sim R^{-1} + F_T \theta^2$ and $g_{4D}^{-2}(\mu \sim T)$ is the SUSY generalization of eqs. (5.1) and (5.2).[9] To be precise, $g_{4D}^{-2}(\mu \sim T)$ used to compute the derivative in eq. (5.3) is the holomorphic gauge coupling (including the topological vacuum angle, i.e. the θ-term), whereas $g_{4D}^{-2}\left(\mu \sim R^{-1}\right)$ in eq. (5.1) is (closer to) the physical or canonical gauge coupling. In general, the canonical gauge coupling differs from the holomorphic gauge coupling due to anomalous Jacobians under the rescaling of gauge fields in going from holomorphic to canonical normalization [10]. Suppose the *massive* gauge KK states (at each level) form $N = 2$ SUSY vector multiplets (as in [4]): in the $N = 1$ SUSY language, these consist of a vector multiplet and a chiral multiplet in the adjoint representation. The anomalous jacobians from these two $N = 1$ SUSY multiplets *cancel* each other [10].[10] This cancelation is related to the fact that there are no corrections to the canonical gauge coupling *beyond* one-loop involving *only massive* gauge KK states (i.e., $N = 2$ SUSY vector multiplets) [10]. The *zero*-modes of the gauge fields form a $N = 1$ SUSY vector multiplet (as usual) which *does* give an anomalous jacobian under the rescaling, and hence the following relation (for an SU(N) gauge group): $1/g_{4D\,c}^2(E) = \mathrm{Re}\left(1/g_{4D\,h}^2(E)\right) - 2N/\left(8\pi^2\right) \ln g_{4D\,c}(E)$ [11, 10], where the subscript c (h) denotes the canonical (holomorphic) gauge coupling. Thus, the *RG scaling or running* of the (real part of) gauge coupling in these two normalizations differs only at *two* (and higher)-loop level. So, the *one*-loop result for the canonical gauge coupling, $g_{4D}^{-2}\left(\mu \sim R^{-1}\right)$, can be generalized to the holomorphic gauge coupling, i.e. to $g_{4D}^{-2}(\mu \sim T)$, by the simple substitution $R^{-1} \to T$ as required by holomorphy. Of

[9] A brief comment on the effect of zero-modes (zero-mode of gauge field and also matter fields) on the gaugino mass is in order here. As mentioned earlier, the running due to zero-modes will result in an additional term $\propto b_0 \ln(\Lambda R)$ in $g_{4D}^{-2}\left(\mu \sim R^{-1}\right)$ and thus seems to give an additional (weak) dependence on T when computing $\partial g_{4D}^{-2}(\mu \sim T)/\partial T$ in the above equation. However, it is clear that this dependence "cancels" when we run from R^{-1} to the weak scale, i.e. the running contribution due to zero-modes (unlike the KK modes) to $g_{4D}^{-2}(\mu \sim \textit{weak scale})$ obviously does *not* depend on R. Of course, the zero-modes do affect the gaugino mass since, at one-loop, $M_{\tilde{g}}/g^2$ is RG-invariant and zero-modes renormalize (as in $4D$) g_{4D} from Λ to the weak scale — this effect on gaugino mass in running from Λ to R^{-1} will appear in the $g_{4D}^2\left(\mu \sim R^{-1}\right)$ term in eq. (5.3) and the RG effect from R^{-1} to the weak scale (not shown here) is the usual ($4D$) running of gaugino mass.

[10] By $N = 2$ supersymmetry, at each massive level, the rescalings, i.e. the wavefunction renormalization, for the chiral and vector multiplets must be the same (up to the loop effect of *zero*-mode gauge fields which do *not* form $N = 2$ SUSY multiplet).

course, the canonical gauge coupling at the cut-off, $g_{4D\,c}^{-2}(\Lambda)$, differs from the holomorphic gauge coupling, $g_{4D\,h}^{-2}(\Lambda) \sim g_{(4+\delta)D\,h}^{-2}(\Lambda)\ T^{\delta}$, by the "one-loop" term from the rescaling anomaly, $\sim -2N/\left(8\pi^2\right)\ln\left(R^{-\delta/2}\right)$ (using $g_{4D\,c} \propto R^{-\delta/2}$); this results in an additional (mild) dependence of the canonical gauge coupling on R.[11]

The low energy ($4D$) gauge coupling depends on R due to two effects. One dependence of $g_{4D}\left(\mu \sim R^{-1}\right)$ on R in eq. (5.1) is from the (one-loop) power-law running as discussed by Kobayashi and Yoshioka (KY) [1]; this is the effect of quantum power-law scaling. This dependence gives

$$\begin{aligned}\frac{\partial g_{4D}^{-2}\left(\mu \sim T\right)}{\partial T}|_{KY} &\approx -\frac{b_{KK}}{8\pi^2}R\left[\hat{V}_{\delta}(\Lambda R)^{\delta}-1\right] \\ &\approx -\delta\ R \times\ \text{running contribution to } g_{4D}^{-2}\left(\mu \sim R^{-1}\right) \\ &\quad \text{(up to a small log-factor, see eq. (5.1))}. \end{aligned} \tag{5.4}$$

Here, increasing R makes $g_{4D}(\mu \sim R^{-1})$ larger due to larger number of KK states contributing to running (with $b_{KK} < 0$) so that the above derivative has positive sign.

The other dependence of $g_{4D}\left(\mu \sim R^{-1}\right)$ on R is from the value of g_{4D} at the cut-off Λ. The $4D$ theory with KK states is derived from the ("fundamental") $(4+\delta)$ theory. Hence, $g_{(4+\delta)D}^{-2}(\Lambda)$ is a fundamental parameter so that (SUSY generalization of) $g_{4D}^{-2}(\Lambda)$ depends on T (see eq. (5.2)) as discussed by Chacko and Luty (CL) [2]. CL use the $5D$ approach in which this dependence is obvious,[12] but it is clear that the same effect appears in the $4D$ approach as well. This effect corresponds to classical power-law scaling in the sense that this dependence of $g_{4D}^{-2}(\Lambda)$ (and hence of $g_{4D}^{-2}\left(\mu \sim R^{-1}\right)$) on R is present in U(1) case also, i.e. it is a tree-level and *not* the running effect. Thus, we get an *additional* contribution

$$\frac{\partial g_{4D}^{-2}\left(\mu \sim T\right)}{\partial T}|b.c. \approx -\delta\ R\ g_{(4+\delta)D}^{-2}(\Lambda)\left(2\pi R\right)^{\delta} \approx -\delta\ R \times g_{4D}^{-2}(\Lambda)\,, \tag{5.5}$$

where b.c. stands for "boundary condition". Since larger R makes $g_{4D}^{2}(\Lambda)$ (and hence $g_{4D}^{2}(\mu \sim R^{-1})$) smaller for fixed $g_{(4+\delta)D}^{2}(\Lambda)$, this contribution to the above derivative is of negative sign.

[11] There are loop corrections to the kinetic terms of matter fields which are on 3-branes (wave-function renormalization Z) and thus there is also an anomalous jacobian under rescaling of matter fields in going to canonical kinetic terms. As in the case of gauge fields, this rescaling modifies the RGE's for gauge couplings only at *two*-loop level and hence does not modify the *one*-loop analysis above. Also, since the matter fields are on 3-branes, the kinetic terms of matter fields do not depend on R at *tree-level* and hence the rescaling anomaly term $\sim 1/\left(8\pi^2\right)\ln Z$ does *not* depend on R at the one-loop level (unlike in the case of rescaling of gauge fields: see above).

[12] $g_{(4+\delta)D}^{-2}(\Lambda)$ is determined by, say, the dilaton field ϕ in $(4+\delta)D$ (in the context of string theory). Thus, $g_{4D}^{-2}(\Lambda)$ depends on a combination of the fields ϕ and radion (R). We can *define* (the real part of) the dilaton chiral superfield S *in* $4D$ to be this combination of ϕ and R [12] so that we get the tree-level expression $g_{4D}^{-2} \sim \mathrm{Re}S$ which is commonly used in the literature.

The contribution to the derivative in eq. (5.5) is always larger (in magnitude) than the one in eq. (5.4) since to keep $g_{4D}^{-2}(\mu \sim R^{-1})$ (eq. (5.1)) positive, i.e. to prevent $g_{4D}(\mu \sim R^{-1})$ from "blowing" up, the running (i.e. loop) contribution to $g_{4D}^{-2}(\mu \sim R^{-1})$ (which enters in eq. (5.4)) has to be smaller in magnitude than the value at Λ (which enters in eq. (5.5)). It is clear from eqs. (5.1), (5.4) and (5.5) that if the suppression due to the loop-factor in the running contribution is compensated by either 1) $g_{4D} \gg 1$ at the cut-off (i.e. $g_{4D}^{-2}(\Lambda)$ is *small*) or by 2) a large number of KK states (i.e. $\Lambda R \gg 1$; in this case, $g_{4D}(\Lambda)$ can be O(1)), then the two contributions to the derivative (and hence to gaugino mass) can be *comparable (in magnitude)*. The first case is ruled out since $g_{4D}(\mu \sim R^{-1})$ (and hence $g_{4D}(\mu \sim \text{weak scale})$) will also be much larger than 1, whereas we know that the measured SM gauge couplings are all at most O(1). In either of these two cases, we see that $g_{(4+\delta)D}^{-2}(\Lambda) \sim O\left(-b_{KK}/(8\pi^2)\Lambda^{\delta}\right) \ll \Lambda^{\delta}$, i.e. the $(4+\delta)D$ theory is strongly coupled at the cut-off Λ — this is expected since from the $(4+\delta)D$ theory point of view, the only way that the running effect can be as important as the tree-level effect is for the theory to be strongly coupled. However, the $4D$ theory can still be weakly coupled (at the cut-off) if $\Lambda R \gg 1$ as discussed above (case 2) and as seen from eq. (5.2).

Reference [1] uses the $4D$ approach, but the tree-level contribution to the gaugino mass (i.e the effect of classical power-law scaling), eq. (5.5), is *not* included — in other words, it is assumed that $g_{4D}^{-2}(\Lambda)$ is a "fundamental" parameter and hence has no functional dependence on R. Then, the running (or quantum power-law scaling) contribution, eq. (5.4), (by itself) gives [1]

$$\begin{aligned} M_{\tilde{g}}(\mu \sim R^{-1})|_{KY} &\approx F_T R \times -\frac{b_{KK}\ g_{4D}^2(\mu \sim R^{-1})}{8\pi^2} \left[\hat{V}_\delta(\Lambda R)^\delta - 1\right] \\ &\approx F_T R \times -\frac{b_{KK}\ g_{4D}^2(\mu \sim R^{-1})}{8\pi^2}\ N_{KK}(\Lambda)\,, \end{aligned} \tag{5.6}$$

where $N_{KK}(\Lambda)$ is the total number of KK states (up to the cut-off).

Whereas, *adding* the contributions in eqs. (5.4) and (5.5) and using eq. (5.1), we get

$$M_{\tilde{g}}(\mu \sim R^{-1}) \approx -\delta F_T R \left[1 + \frac{b_{KK}}{8\pi^2} g_{4D}^2\left(\mu \sim R^{-1}\right)\left((\hat{V}_\delta - 1)/\delta + \ln(\Lambda R)\right)\right]. \tag{5.7}$$

It is clear from the above discussion that the KY result in eq. (5.6) is smaller than (or at most comparable to, as argued above) the above result and also of opposite sign. In any case, it is clear that according to KY, the gaugino mass is a *loop-level* effect (which, however, can be *enhanced* by a large number of KK states), whereas CL argue that it is (also) a *tree-level* effect.

In the U(1) case with matter fields on 3-brane, the discrepancy between the two results is even more obvious. According to KY, e_{4D} does *not* depend on T (since there is no running due to KK states: $b_{KK} = 0$) so that there is *no* photino mass (at one-loop), whereas CL show that there *is* photino mass: in the $4D$ approach, this is due to the dependence of $e_{4D}^{-2}(\Lambda)$ on T. To repeat, the resolution of this discrepancy (which also applies to the non-abelian case) is that the boundary (or tree-level) conditions in the two cases are different — KY assume that $g_{4D}^{-2}(\Lambda)$ is fundamental (no functional dependence on R), whereas CL assume $g_{(4+\delta)D}^{-2}(\Lambda)$ is fundamental and hence $g_{4D}^{-2}(\Lambda)$ *does* depend on R (eq. (5.2)).

Actually, CL did not *explicitly* include (one-loop) power-law running of $g_{(4+\delta)D}$ (or equivalently that of g_{4D}), i.e. the effect of quantum power-law scaling — this corresponds to assuming $b_{KK} = 0$ in our notation. So they get $M_{\tilde{g}}(\mu \sim R^{-1})|_{CL} \approx -\delta F_T R$ [2] (i.e. eq. (5.7) with $b_{KK} = 0$). This is of course valid for U(1) case. In the non-abelian case, as argued earlier, it is possible that the running (loop) contribution to gaugino mass (eq. (5.4)) can be of the *same order* as the tree-level contribution (eq. (5.5)) if the $(4+\delta)D$ theory is strongly coupled at Λ. However, in the non-abelian case, we can see that the expression for the gaugino mass is approximately the same (as in the U(1) case) since (with $b_{KK} \neq 0$) the second term in the bracket in eq. (5.7) (including the $\ln(\Lambda R)$ piece) is small. This is true even if $(\Lambda R)^{\delta} \sim 1/\text{loop-factor} \gg 1$, which, as discussed in section 4, corresponds to the maximum value of Λ. Here, we assume $g_{4D}\,(\mu \sim R^{-1}) \lesssim \mathrm{O}(1)$ as in the MSSM. Explicitly, with $b_{KK} \neq 0$, it is obvious that in eq. (5.3) there are extra terms (as compared to U(1) case) in *both* $g_{4D}^{-2}(\mu \sim R^{-1})$ (see eq. (5.1)) and $\partial g_{4D}^{-2}(\mu \sim T)/\partial T$ (eq. (5.4)) which (almost) *cancel* each other and give the same expression for the gaugino mass as in the U(1) case.[13]

6. Summary

In summary, we have reviewed the equivalence of the "$4D$ with KK states" and "$5D$ with matching" approaches to study theories with gauge fields in extra dimensions. We reiterated that there are two different power-law scalings for effective gauge couplings which we referred to as classical and quantum.

In supersymmetric theories with SUSY breaking in the radion, the $4D$ approach *appears* to give gaugino mass at loop-level [1], whereas the $5D$ approach gives gaugino

[13]In other words, for $\Lambda R \gg 1$, we get from eqs. (5.1) and (5.2) $g_{4D}^{-2}\left(\mu \sim R^{-1}\right) \approx (2\pi R)^{\delta}\,[g_{(4+\delta)D}^{-2}(\Lambda) + \hat{V}_{\delta}/(2\pi)^{\delta}\ b_{KK}/\left(8\pi^2\right)\Lambda^{\delta}/\delta]$ so that $g_{4D}^2\ \partial g_{4D}^{-2}/\partial T \approx -\delta R$ in eq. (5.3) and hence $M_{\tilde{g}}(\mu \sim R^{-1}) \approx -\delta F_T R$ (which is the expression in [2]), *irrespective* of the values of b_{KK} or $g_{(4+\delta)D}(\Lambda)$. In fact, assuming $g_{4D}(\Lambda) \sim \mathrm{O}(1)$ (as in the MSSM), this corresponds to case 2 mentioned after eq. (5.5) — the tree- and loop-level terms in the above expression for $g_{4D}^{-2}\left(\mu \sim R^{-1}\right)$ are *comparable*. The other possibility is that $\Lambda R \sim \mathrm{O}(1)$ so that R^{δ} *cannot* be "factored out", unlike in the case $\Lambda R \gg 1$ (of course, in this case, the result in eq. (5.1) may not be valid as mentioned before). In this case, if the loop correction to the gauge coupling is comparable to the value at the cut-off, then we get $g_{4D}^2(\Lambda)$ $\left(\text{and also } g_{4D}^2\left(\mu \sim R^{-1}\right)\right) \gg 1$ (corresponding to case 1 mentioned before). This case *cannot* correspond to the MSSM, where all gauge couplings are O(1). Therefore, if $\Lambda R \sim \mathrm{O}(1)$ in the MSSM, then the loop correction to the gauge coupling (and hence the running contribution to gaugino mass) has to be *small* so that we again recover the CL result.

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mass at tree-level [2]. We clarified that this discrepancy is due to the fact that, even in the $4D$ approach, there *is* a tree-level contribution to gaugino mass due to the boundary condition, eq. (5.2), where $g^{-2}_{(4+\delta)D}(\Lambda)$ is a fundamental parameter and hence $g^{-2}_{4D}(\Lambda)$ depends on R. This contribution was *not* included in [1] and it corresponds to the tree-level effect in [2]. We also showed that even if the loop contributions to gauge coupling (and to gaugino mass) analyzed in [1] (and which are *not* included in the *tree-level* analysis of [2]) are enhanced by large number of KK states in the $4D$ approach (or equivalently, due to strong coupling in $5D$ approach), the "loop correction" to the (tree-level) *expression* for gaugino mass given in [2] is *small*.

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